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# Considering the Effects of Local Uplift and Soil Elastic Perfectly Plastic Behavior on Soil-Structure Interaction Problem under Seismic Loading

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## ABSTRACT

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# CONSIDERING THE EFFECTS OF LOCAL UPLIFT AND SOIL ELASTIC PERFECTLY PLASTIC BEHAVIOR ON SOIL-STRUCTURE INTERACTION PROBLEM UNDER SEISMIC LOADING

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## Abstract

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**Keywords:** Elastic-perfectly Plastic, Local Uplift, Soil-Structure Interaction

## 1. Introduction

In this work, seismic response of an interacting soil-structure system is studied by taking into account elastic perfectly plastic soil behavior and local uplift of the foundation. The equations governing the problem are derived by taking into account the equilibrium of the coupled soil-structure system. Small rotations are assumed and the mat of the structure is idealized as a single-degree-of-freedom (SDOF) oscillator while the foundation is taken to be rigid. The obtained discrete system has three degrees-of-freedom which are related to mat tip lateral displacement, base vertical displacement and base rotation. The associated non-linear system of ordinary differential equations is integrated by using the Matlab command ode15s. A parametric analysis is then performed in order to evaluate the maximum response of the system under seismic loading as function of the chosen soil model and key

dimensionless parameters. These include the slenderness ratio and the ratio of mat elastic resonance frequency to the vertical resonance frequency of foundation.

## 2. Literature Review

In the field of earthquake engineering, soil-structure interaction (SSI) is an important phenomenon that has to be taken into account in order to improve description of the coupled structure foundation system behavior. Many authors have studied this problem and various approaches based on analytical models, rigid bodies or flexible systems were introduced to deal with the different aspects resulting from this particular interaction [1-3].

The Winkler foundation based models consider that the soil can be represented by distributed elastic springs [4-10]. These models which are simple and computationally efficient were proved to be

effective for the simulation of soil-structure interaction for common civil engineering structures that are undergoing relatively small displacements. Some nonlinear effects arise when the structure has the possibility to experience a rocking rigid block motion which can yield then to foundation base uplifting.

Numerous analytical studies of the seismic response of an elastic-plastic oscillator have assessed the beneficial effect of SSI on the system response. These include studies dealing with rigid foundation base [11-13] and flexibly-supported systems [14-16]. Plastic behavior of the soil was integrated in [16] where the authors have investigated the seismic response of a single degree-of-freedom system of bridge piers. The interaction between bridge piers and supporting soil was simulated by an improved elastic-plastic Winkler model in which the uplifting effect of foundations was considered. They have concluded that permitting uplifting and plasticity of foundations in design can reduce the seismic load acting on the pier body and play thus an advantageous role in the seismic capability of bridge piers.

Allotey et al. [18] examined the strengths and limitations of the FEMA 356 nonlinear Winkler modeling approach, when used in predicting the cyclic response of various recent soil-footing-structure interaction experiments. The results of the study show that the model is capable of satisfactorily predicting all the different cyclic response quantities apart from permanent horizontal displacements.

Zheng et al. [19] studied a two-degrees-of-freedom model of reinforced concrete continuous bridge with foundation-pier dynamic interaction. They have considered nonlinearities of both reinforced concrete pier and soil foundation in order to evaluate seismic response of this system. The authors have based their modeling on the revised Winkler foundation model with

both plasticity and local uplift effects. They have used the modified Takeda hysteretic model to represent soil behavior and considered four ground motions to assess the seismic performance of the bridge-foundation system.

### 3. Research Methods

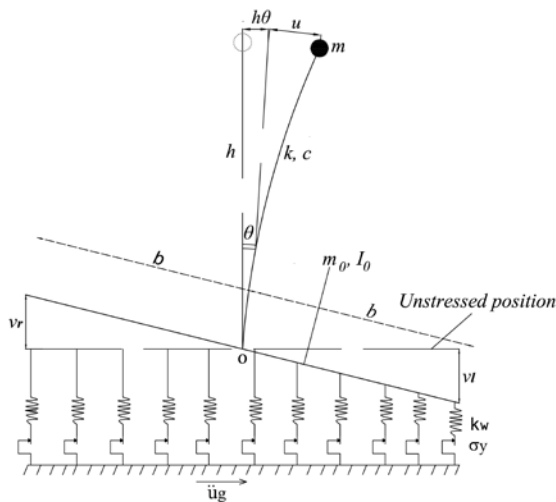
#### 3.1 Description of the model

The system considered is shown in Fig.1. The structure is assumed to behave as a beam like mat which can be characterized by its first mode of vibration. This yields to a SDOF oscillator which is characterized by its natural frequency  $\omega = \sqrt{k/m}$  and damping ratio  $\xi = c/(2m\omega)$  where  $k$  is the lateral stiffness,  $m$  the mass and  $c$  the lateral damping. The oscillator is placed at height  $h$  from the base.

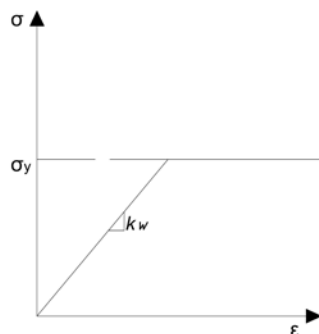
The mat is supposed to be mounted on a rigid foundation basis that is assumed to react as a rigid rectangular plate of negligible thickness. The foundation mass denoted  $m_0$  is taken to be uniformly distributed; the width of the base is  $2b$  and the total moment of inertia about the center of the foundation is denoted by  $I_0$ . In the following  $M_r$  designates the total moment acting on the base mat,  $u$  the elastic horizontal displacement of the mat tip relative to the base and  $\ddot{u}_{gx}$  the seismic acceleration. The following notations are also used:  $\omega_v = \sqrt{k_w/m_0}$  vertical vibration frequency of foundation;  $\alpha = h/b$  slenderness ratio;  $\beta = \omega_v/\omega$  frequency ratio;  $\gamma = m/m_0$  foundation mass to superstructure mass ratio.

The soil-structure interaction takes place at the interface separating the rigid footing and the foundation soil. The coupling of the system is modeled by Saint-Venant's units that are mounted in parallel. Each unit consists of an elastic spring connected in series with a plastic element which is activated if the strain from elastic spring

reaches the yield stress  $\sigma_y$ . This last corresponds to the plasticity plateau as shown on Fig. 2 and where  $k_w$  is the soil modulus of elasticity per unit length. The units are distributed over the entire width of the foundation, see Fig. 1. Horizontal slippage between the mat and supporting elements is assumed to be negligible.



**Figure 1. Flexible structure resting on elastic perfectly plastic foundation.**



**Figure 2. Elastic perfectly plastic behavior of spring and plastic element mounted in series.**

In the elastic range of deformation, the stress-strain relationship is given by:  $\sigma = k_w(v \pm x\theta)$  where  $v$  is the vertical displacement of the base mat centre of gravity,  $\theta$  is the angle of rotation of the mat base and  $x$  is the abscissa along the foundation with origin taken at its centre. When the stress reaches the threshold  $\sigma_y$  for some plastic frontier point  $x_{pi}$ , where

$i=l$  or  $i=r$  that designates left or right edge, then  $\sigma = \sigma_y$ . Also local uplifting occurs if  $\sigma = 0$  for some critical position point  $x_{ci}$ .

Fig.3 presents the overall scheme adopted for performing the elastic perfectly plastic analysis and dealing with the plasticity and uplifting nonlinearities. The steps in the scheme are as follows:

Step 1: under a selected ground motion and for actual positions of base that are associated to uplift or plastic frontier conditions, compute the answer of the coupled system for the given increment and obtain: (i) rotation of the base, (ii) vertical displacement, (iii) horizontal displacement.  
Step 2: Compare the actual obtained value of  $\theta$  with zero.

Step 3: Determine the actual value of critical positions  $x_{ci}$  which corresponds to  $\sigma_y = 0$ .

Step 4: Determine the actual value of plastic positions  $x_{pi}$  which corresponds to  $\sigma_y = k_w(v \pm x_{pi}\theta)$ .

Step 5: Compare  $x_{pi}$  with algebraic value of half width of foundation mat.

Step 6: Compare  $x_{pi}$  with  $x_{ci}$ .

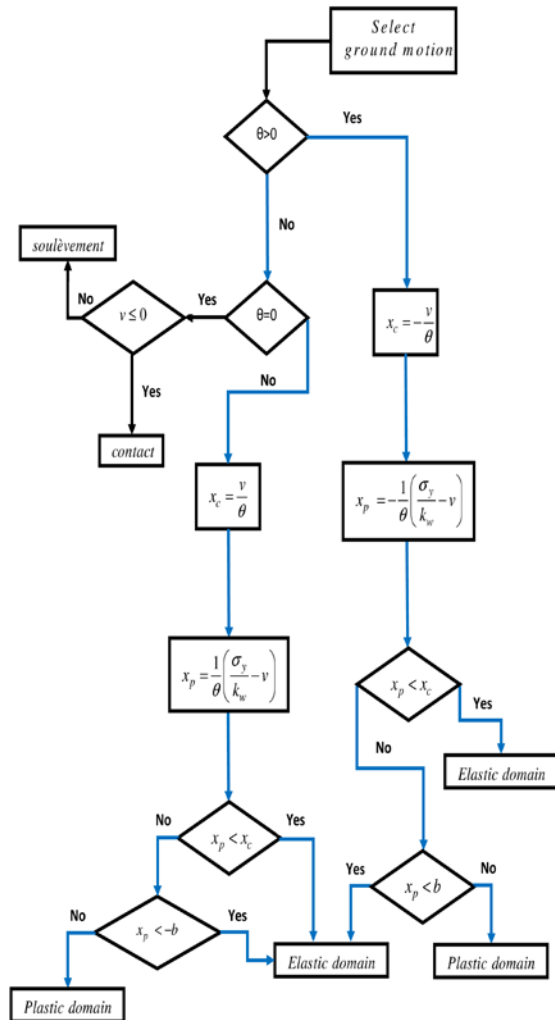
### 3.1 Equations of motion

The equations of motion can be derived by taking into account the equilibrium of the coupled soil-structure system. The following equilibrium equations can be written:

- Equilibrium of forces acting in the horizontal direction:  $\sum F_x = 0$
- Equilibrium of forces in the vertical direction:  $\sum F_y = 0$
- Equilibrium of moments about the center of the foundation of the mat:  $\sum F_z = 0$

Because the rotations are assumed to be small, the following approximations hold  $\cos(\theta) \approx 1$  and  $\sin(\theta) \approx \theta$  (1)

Considering the equilibrium of forces in the lateral direction, the simplified equation of motion in terms of the mat tip lateral displacement writes



**Figure 3. Schematic outline of the elastic perfectly plastic analysis procedure**

$$m\ddot{u} + mh\ddot{\theta} + c\dot{u} + ku = -m\ddot{u}_{gx}(t) \quad (2)$$

The equilibrium of forces in the vertical direction can be written as follows:

$$(m + m_0)\ddot{v} + F_v = -(m + m_0)g \quad (3)$$

with

$$F_v = \int_{-b}^{x_{pl}} k_w v_{(\ell,r)} dx + \int_{x_{pr}}^b k_w v_{(\ell,r)} dx \quad (4)$$

$$+ \int_{x_{pr}}^b k_w v_{(\ell,r)} dx$$

The vertical displacement at the edges of the foundation mat, Fig. 1, measured from

the initial unstressed positions are:

$$v_i = v \pm x \theta, \quad i = l, r \quad (5)$$

In equation (4),  $F_v$  is the total vertical force acting on the base mat. This force is obtained as

$$F_v = \int_{-b}^{x_{pl}} k_w (v \pm x \theta) dx + \int_{x_{pr}}^b k_w (v \pm x \theta) dx \quad (6)$$

$$+ \int_{x_{pr}}^b k_w (v \pm x \theta) dx$$

Because the foundation cannot extend above its initial unstressed position, an edge of the foundation mat would uplift at the time instant for which the following condition is fulfilled [19]

$$v_i(t) > 0, \quad i = l, r \quad (7)$$

Integrating equation (6) by taking into account the uplifting condition given by equation (7), one arrives after simplifications at the following expression

$$F_v = k_w v (x_{pl} + 2b + x_{pr} - x_{pl} - x_{pr}) + \varepsilon_2 k_w \theta \left( -\frac{1}{2} \varepsilon_1^2 b^2 + b^2 \right) + \varepsilon_2 k_w \theta \left( \frac{1}{2} x_{pl}^2 + x_{pr}^2 - \frac{1}{2} \varepsilon_1^2 x_{pl}^2 - \frac{1}{2} \varepsilon_1^2 x_{pr}^2 \right) \quad (8)$$

with

$$\varepsilon_1 = \begin{cases} 1 & \text{contact at both edges} \\ \frac{\varepsilon_2 v}{b\theta} & \text{one edge is uplifted} \end{cases} \quad (9)$$

$$\varepsilon_2 = \begin{cases} -1 & \text{left edge uplifted} \\ 0 & \text{contact at both edges} \\ 1 & \text{right edge uplifted} \end{cases} \quad (10)$$

Substituting equation (8) into equation (3), one obtains the following equation:

$$\ddot{v} = -\frac{\varepsilon_2 k_w \theta}{m+m_0} \left( \frac{1}{2} x_{pl}^2 + x_{pr}^2 - \frac{1}{2} \varepsilon_1^2 x_{pl}^2 - \frac{1}{2} \varepsilon_1^2 x_{pr}^2 \right) - \frac{\varepsilon_2 k_w \theta}{m+m_0} \left( -\frac{1}{2} \varepsilon_1^2 b^2 + b^2 \right) - \frac{k_w v}{m+m_0} (x_{pl} + 2b + x_{pr} - x_{pl} - x_{pr}) - g \quad (11)$$

Taking the resultant moment about the centre of the foundation at the base of the mat, the following equation is readily obtained:

$$I_0 \ddot{\theta} + hm(h\ddot{\theta} + \ddot{u} + \ddot{u}_{gx}) + M_r = 0 \quad (12)$$

with  $I_0 = m_0 b^2 / 3$  and where  $M_r$  is the resistant moment which represents the global action of spring and plastic slider system acting on the foundation base. It is derived by considering the forces applied on the free body diagram of the base mat as

$$M_r = k_w \int_{-b}^{x_{pl}} [\pm vx \pm x^2 \theta] dx + k_w \int_{x_{pl}}^{x_{pr}} [\pm vx \pm x^2 \theta] dx + k_w \int_{x_{pr}}^b [\pm vx \pm x^2 \theta] dx \quad (13)$$

Integrating equation (13) and taking into account equation (7), one obtains

$$M_r = \varepsilon_2 k_w \left( \frac{1}{2} vx_{pl}^2 + \frac{1}{3} \theta x_{pl}^3 - \frac{1}{2} \varepsilon_1^2 vb^2 + \frac{1}{3} \theta b^3 + \frac{1}{2} vx_{pr}^2 + \frac{1}{3} \theta x_{pr}^3 - \frac{1}{2} \varepsilon_1^2 vx_{pl}^2 - \frac{1}{3} \varepsilon_1^2 \theta x_{pl}^3 + \frac{1}{2} vb^2 + \frac{1}{3} \theta b^3 - \frac{1}{2} \varepsilon_1^2 vx_{pr}^2 - \frac{1}{3} \varepsilon_1^2 \theta x_{pr}^3 \right) \quad (14)$$

Substituting equation (14) into equation (12) yields the following equation

$$\ddot{\theta} = \frac{3\alpha^2}{\gamma h} (c\dot{u} + ku) - \frac{3\varepsilon_2 \alpha^2 k_w}{m\gamma h^2} \frac{1}{2} vx_{pl}^2 + \frac{1}{3} \theta x_{pl}^3 - \frac{1}{2} \varepsilon_1^2 vb^2 + \frac{1}{3} \theta b^3 + \frac{1}{2} vx_{pr}^2 + \frac{1}{3} \theta x_{pr}^3 - \frac{1}{2} \varepsilon_1^2 vx_{pl}^2 - \frac{1}{3} \varepsilon_1^2 \theta x_{pl}^3 + \frac{1}{2} vb^2 + \frac{1}{3} \theta b^3 - \frac{1}{2} \varepsilon_1^2 vx_{pr}^2 - \frac{1}{3} \varepsilon_1^2 \theta x_{pr}^3 \quad (15)$$

Finally, the equations of motion of the system are formed by equations (2), (11) and (15). This is a highly nonlinear system of second order ordinary differential equations. Its integration can be achieved iteratively by using the Matlab command ode15s. This enables then to calculate the response of the structure and to perform parametric studies.

## 4. Analysis & Results

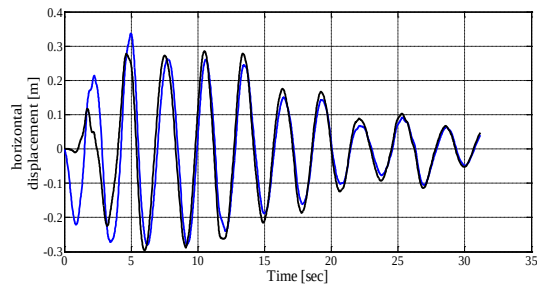
### 4.1 Response to seismic excitation

In the following, the systems responses for both elastic and perfectly plastic soil models are calculated. The system response is considered under the 1940 El Centro earthquake record input and with fixing the following dimensionless parameters during the simulations:

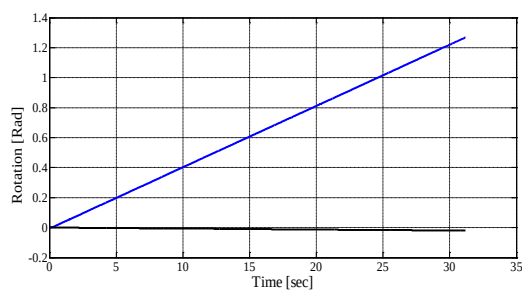
$\alpha=8, \beta=2, \gamma=0.1, \zeta=0.05, \zeta_v=0.03$ . The other parameters were fixed at the following values:  $h=12m, m=105.07 \times 10^3 kg, c=21.85 \times 10^3 kg.s^{-1}, \omega=2.08Hz, \sigma_y=1.5 \times 10^{-4} MPa$  and  $k_w=18.4 \times 10^6 N/m$ .

Fig. 4 gives a comparison of the seismic response of the system in terms of the lateral displacement of the structure and the foundation rotation, for both elastic and perfectly plastic analyses.

The perfectly plastic behavior was found to moderate the system response: it decreases the tip displacement in comparison with the elastic analysis. The same observation can be done with regards to rotation. One can notice also from Fig. 4b that rocking has occurred for the soil elastic model while this is absent in the case of elastic-perfectly plastic soil.



**Figure 4a. Horizontal displacement of the structure: blue color is for elastic analysis black corresponds to the perfectly plastic analysis (  $\alpha=8, \beta=2, \gamma=0.1, \zeta=0.05, \zeta_v=0.03$  )**



**Figure 4b. Horizontal displacement of the structure: blue color is for elastic analysis black corresponds to elastic perfectly plastic analysis (  $\alpha=8, \beta=2, \gamma=0.1, \zeta=0.05, \zeta_v=0.03$  )**

#### 4.2 Parametric investigation of maximum response

A parametric study was conducted in order to analyze the effect of the chosen soil model and as function of the key parameters  $\alpha, \beta, \gamma$ . The maximum values of the tip mat displacement and base rotation are considered when elastic and elastic perfectly plastic soil interacting models are considered along with a base-mat foundation which is allowed to undergo local uplifting. The response of the system under the 1940 El Centro earthquake record input was calculated for both of these soil models.

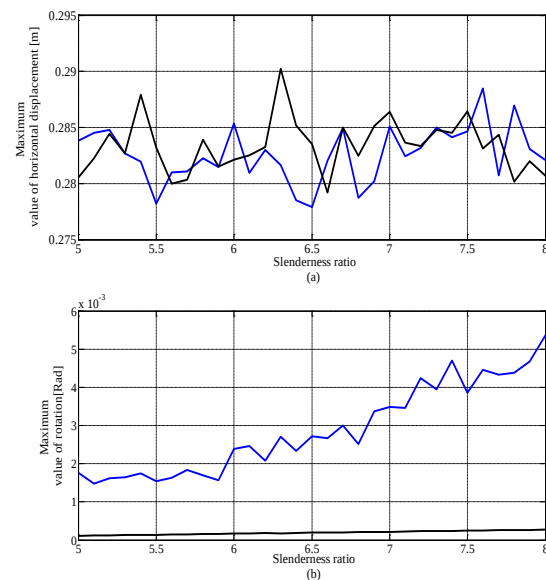
Two parametric studies were performed:

- The first one concerned variation of the maximum values of the mat tip displacement and base rotation as function

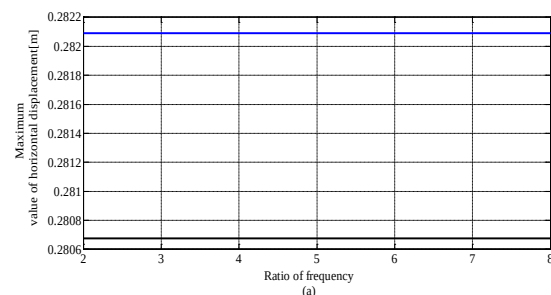
of the slenderness ratio with  $\beta=2$ . Fig. 5 gives the obtained results for  $\alpha \in [5, 8]$ .

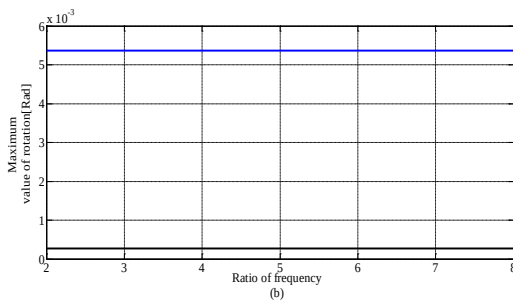
- In the second study, the maximum response is evaluated as function of the frequency ratio with  $\alpha=8$ . Fig. 6 gives the obtained results for  $\beta \in [2, 8]$ .

As shown in Fig.5, the maximum values of the tip displacement and base rotation vary largely with  $\alpha$ . The perfectly plastic analysis moderates the magnitudes of rotation. However this behavior does not moderate all the time the magnitudes for the tip displacement. The magnitude can be increased or decreased depending on the domain containing the slenderness ratio.



**Figure 5. Maximum values of the tip lateral displacement and base rotation as function of the slenderness ratio  $\alpha$  : (a) Tip displacement; (b) base rotation; blue color is for elastic analysis and black corresponds to elastic perfectly plastic analysis**





**Figure 6. Maximum values of the tip lateral displacement and base rotation as function of the frequency ratio  $\beta$  : (a) Tip displacement; (b) base rotation; blue color is for elastic analysis and black corresponds to the elastic perfectly plastic soil**

## 5. Discussion & Conclusion

In this work, a parametric analysis was performed in order to evaluate the maximum response of the soil-structure system under the 1940 El Centro earthquake record input by taking into account local uplift of the foundation and soil plastic behavior. This was conducted in terms of key dimensionless parameters that include the slenderness ratio and the ratio of mat elastic resonance frequency to the vertical resonance frequency of foundation. The obtained results have shown that the structure response is largely affected by these factors in the investigated ranges of parameters. Taking into account local uplift of the foundation under seismic loading and the plastic regime of strains was found to be beneficial for the structure because it yields smaller rotational angles. The tip horizontal displacement can be increased or decreased by considering plasticity of soil depending on the considered range of parameters.

## 6. References

- [1] Paolucci, R. (1997), Simplified evaluation of earthquake induced permanent displacements of shallow foundations, *Journal of Earthquake Engineering* (3): pp. 563-579.
- [2] Gottardi, G., Housby, G. T., Butterfield, R. (1999), Plastic response of circular footings on sand under general

planar loading, *Géotechnique Title of the journal* (49): pp. 453-469.

- [3] Paolucci, R., Shirato, M., Yilmaz, M.T. (2008), Seismic behaviour of shallow foundations: Shaking table experiments vs numerical modelling, *Earthquake Engineering and Structural Dynamics* (37): pp. 577-595.

- [4] Cremer, C., Pecker, A., Davenne, L. (2002), Modelling of nonlinear dynamic behavior of a shallow strip foundation with macro-element, *Journal of Earthquake Engineering* (2): pp. 175-211.

- [5] Chatzigogos, C.T., Pecker, J., Salencon, J. (2009), Macroelement modeling of shallow foundations, *Soil Dynamics and Earthquake Engineering* (29): pp. 765-781.

- [6] Grange, S., Kotronis, P., Mazars, J. (2008), A macro-element for a circular foundation to simulate 3D soil-structure interaction, *International Journal of Numerical and Analytical Methods in* (32): pp. 1205-1227.

- [7] Grange, S., Kotronis, P., Mazars, J. (2009), A macro-element to simulate 3D soil-structure interaction considering plasticity and uplift, *International Journal of Solids and Structures* (46): pp. 3651-3663.

- [8] Grange, S., Kotronis, P., Mazars, J. (2009), A macro-element to simulate dynamic Soil-Structure Interaction, *Engineering Structures* (31): pp. 3034-3046.

- [9] Grange, S., Botrugno, L., Kotronis, P., Tamagnini, C. (2011), The effects of Soil-Structure Interaction on a reinforced concrete viaduct, *Earthquake Engineering and Dynamics Structural* (40): pp. 93-105.

- [10] El Abbas, N., Khamlichi, A., Bezzazi, M. (2016), Seismic response of foundation-mat structure subjected to local uplift,

*Coupled Systems Mechanics* (6): pp. 113-132.

[11] Newmark, N., and Hall, W.(1973), Seismic Design Criteria for Nuclear Reactor Facilities, *Practices for Disaster Mitigation, National Bureau of Standards, U.S. Department of Commerce, Report* (46): pp. 209–236.

[12] Riddell, R., and Newmark, N.(1979), Statistical Analysis of the Response of Nonlinear Systems Subjected to Earthquakes, *Dept. of Civ. Engrg., University of Illinois, Urbana, Structural Research* (468).

[13] Hidalgo, P. A., Arias, A.( 1990), New Chilean code for earthquake-resistant design of buildings, *Proceedings, 4th U.S. Nat. Conf. Earthquake Engrg, Palm Springs, Calif.,* (2): pp. 927–936.

[14] Ciampoli, M. , Pinto, P. E. (1995) ,Effects of Soil-Structure Interaction on Inelastic Seismic Response of Bridge Piers, *Journal of Structural Engineering, ASCE,* (121): pp. 806- 814.

[15] Housner, G. W.( 1963), The behavior of inverted pendulum structures during earthquakes,*Bull. Seismol. Soc. Am.*(53) : pp.403–417

[16] Priestley, M. J. N. Park, R. (1987),Strength and Ductility of Concrete Bridge Columns Under Seismic Loading, *ACI Structural Journal,* (84): pp. 61-76.

[17] Chen, X.C., Lai, J. (2003), Seismic response of bridge piers on elasto-plastic Winkler foundation allowed to uplift, *Sound and Vibration* (266): pp. 957-965.

[18] Allotey, N., El Naggar, M. H. (2008), An investigation into the Winkler modeling of the cyclic response of rigid footings, *Soil Dynamics and Earthquake Engineering* (28): pp. 44–57.

[19] Yue, Z., Bo, C., Weizhen, C. (2015), Elasto-Plastic Seismic Response of RC Continuous Bridge with Foundation-Pier Dynamic Interaction, *Advances in Structural Engineering* (18).